A practical microtonal pitch space; theoretical and psychoacoustic issues, compositional applications

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This paper describes a pitch space representation of a microtonal system that:
• is rooted in contemporary understanding of the sensory procedures involved in pitch perception
• functions within the confines of the "standard" 12 semitone equal temperament (ET) system ubiquitous in Western music by treating ET as a functional abstraction of the important microtonal relationships that exist between pitches
• allows microtonal intonation to show the pitch hierarchy(ies) active in a passage, clarify the function of each note in the current hierarchy and denote the course (origin and goal) of a progression
• gains for composers the ability to make systematic use of microtonal information to enhance the expressivity of a musical work

The paper closes with music examples of applications of the above pitch space in composition.

Using existing knowledge to define a practical microtonal system

Structural, systematic use of microtones is widely found in eastern music. For example, in Indian, Arab and Byzantine music microtones are part of the structural vocabulary and are used systematically to enhance expressivity. In the Western tradition we also use microtonal variations systematically as part of the expressiveness of everyday speech. In fact, "the specific and highly structured patterns of Western speech intonation carry much of the meaning of speech." [O'connor and Arnold 1973]

To be sure, microtonal variations are also present in Western equal temperament music. Performances of Western classical music have been found to include a large variety of tunings for a given interval [Ward 1970]. It is interesting to note that all listeners and musicians involved in the analyzed performances "agreed that the compositions were performed correctly" [Burns and Ward 1978]. This assessment might be considered strange as the equal temperament system, in theory, does not allow the use of microtones. We would therefore expect that listeners would assess performances including microtonal variations as incorrect. Burns and Ward [1978] show that listeners' tolerance for these microtonal variations can be partly attributed to the effects of categorical pitch perception resulting from acculturation to the 12 semitone equal temperament tuning system (from now on 12ET). The 12ET system makes no systematic use of microtonal variations. In this context, sensitivity to pitch intonation only has to go as far as being able to place each note of a work in one of the 12 semitone categories. Thus western listeners may choose to ignore microtonal intonations that lie well within one of the twelve categories and be sensitive only to microtonal variations that lie near the borders of two semitone categories (close to the quartertone intonation – 50 cents above or below a semitone).
Just because western listeners may choose to ignore microtonal variations in a 12ET performance it does not mean that western listeners can not hear microtones. We said that we use microtones consistently in our speech and we know that most of us have no problem appreciating performances of eastern microtonal music. There are even examples of listeners being sensitive to microtonal variations in performances of standard western literature when these microtonal intonations are used consistently [Makeig 1982, Boomsliter and Creel 1963].

So how does a listener decide whether to pay attention to microtonal intonations or not? This paper will reinforce by concrete examples the notion that context is the key to the level of microtonal sensitivity used by the listener. As long as the composition being performed uses microtones in a structural, perceptually viable, systematic manner then the listener will immediately pick up on this information and will quickly develop the microtonal sensitivity necessary for the appreciation of the work. Therefore, inclusion of perceptually viable microtonal intonations at the structural level of the pitch vocabulary being used and facilitation of systematic, consistent use of those microtones in composition should be the main concerns for the creation of a microtonal system.

The paper aims at the creation of a microtonal system that addresses the above points and at the same time is practical. A practical microtonal system should go further than perceptual viability; the microtonal vocabulary should be easily perceivable. The rapid development of psychoacoustics allows us to establish a solid hierarchy of intervallic relationships based on their prominence/importance in pitch perception processes. We will trace this hierarchy and show that it promotes the use and facilitates the perception of specific microtonal variations. We will base our microtonal vocabulary on those variations.

Another practical concern is that we want this microtonal system to be part of music making in the west. Recognizing that the 12 semitone equal temperament system (12ET) is the predominant system in the western world, the paper will show that structural, easily perceivable, systematic use of microtones can actually take place within the confines of the 12ET system. For this to happen, we need to use microtones that are a natural, organic extension of the 12ET system. The underlying structure of the 12ET system has strong ties to the pitch perception based hierarchy of intervallic relationships and consequently to the microtones that this hierarchy promotes. However, because of practical and functional issues ET abstracted this microtonal information. We will show that a great part of this microtonal information can be added back to the system without influencing its functionality. In fact, the microtonal information added will enhance the structures, functions and expressive capabilities of 12ET. We will further show that through minimal exposure to this microtonally enhanced 12ET system the ear can begin to pick up the resulting microtonal structures. Twelve semitone categorical perception can be quickly refined to include perception of these structural microtonal variations.

A final practical concern is that the suggested microtonal system should be easy to use in composition. We know that we will be basing the system on a hierarchy of intervallic relations. We have suspected for centuries, and recent perception research (Terhardt,
Parncutt, Lerdahl, Krumhansl, Bharucha) that will be discussed in this paper has verified beyond doubt, that the hearing of a pitch causes the instant activation of a mental representation of a hierarchy of pitch/intervallic relations. Therefore, a hierarchy of pitch/intervallic relations should be represented as a pitch space that allows the composer to see with one glance the complete structure of the hierarchy activated by each pitch and also the complete set of relations between multiple activated hierarchies in the case of simultaneities. An easy to use microtonal pitch system will be one that can be represented as a compact, finite pitch space that allows the composer to see at once all active relationships. We will see that a P5s/M3s pitch space is the ideal representation of the pitch perception based hierarchy of intervallic relationships. Since this hierarchy serves as the underlying structure of both the microtonal vocabulary of our system and of 12 note ET, a P5s/M3s space is the ideal representation for embedding our microtonal system. We will see that a P5/M3s pitch space can incorporate the complete microtonal vocabulary of our system and still remain compact and finite.

The pitch perception based hierarchy of intervallic relationships and its pitch space representation

We will the start the building of the pitch perception based hierarchy of intervallic relationships from the simplest, most prime, pitch perception instance: the perception of the pitch of a single note.

When we say that we hear a single complex tone we are actually hearing a number of simultaneous simple tones since a complex tone is made of up a number of simple tone harmonics. Extensive testing by Plomp [1964] has shown that in the frequency areas where most musical useful pitches lie (C2 to C7) the number of audible, distinguishable harmonics varies from the first 8 (for low pitches) to the first 5 (for high pitches). For his pitch perception model, Terhardt [1979, 1982] calculates that harmonics up to the 11th might be audible. Parncut [1989, p.137] makes a convincing argument that it is more practical and realistic to assume that for “isolated musical tones in the central musical pitch range harmonics 1 to 10 are audible”. Since throughout this paper we will be dealing with octave generalized pitch spaces, we will assume octave equivalence from this point on. The octave generalized frequencies represented by the first 10 harmonics of a complex tone form a collection where the fundamental frequency, its fifth above (P5) and its major third above (M3) are most prominent. The fundamental pitch appears in this collection four times (1st, 2nd, 4th, 8th harmonics), its perfect fifth two times (3rd, 6th harmonics), its major third two times (5th, 10th harmonic) and its minor 7th and major 9th only once. Ritsma [1967] has shown that the lower a harmonic, the more dominant its role in the perception of the pitch of a complex tone. The lowest harmonics, and thus the most prominent in pitch perception, are the fundamental (and its 1st and 2nd octaves) followed by its P5 and M3 above. Ritsma has also shown that for tones in the range of speech and in the central musical range (100 to 400 Hz) only the 3rd, 4th and 5th harmonics play an important role in pitch perception. The 3rd, 4th and 5th harmonics, octave generalized, form again a collection made up of the fundamental and its P5 and M3 above.
We can thus venture to say that if the fundamental is the strongest pitch sensation created by a harmonic complex tone then the second most prominent pitch sensation created by a harmonic complex tone is a major triad build on the fundamental (fundamental + its M3 above + its P5 above).

This means, that whenever a pitch is heard its fifth and major third above are also implicitly heard. This creates a very strong bond, a very strong relationship, between a note and its fifth and major third above. This partially explains our frequent expectation for those notes to be heard together or immediately after each other. By taking into account the microtonal intonations of the harmonics that create these strong relationships (the intonations of harmonics 1,2,3,4,5,6,8,10) we can define with great accuracy the microtonal intonations of these intervallic relationships. The M3 should be 386 cents and the P5 702 cents. Those are also the just intonations (JI) of these primary intervals. To give a specific example of these relationships we can say that the note C is most strongly related to E-14 cents and G+2 cents (from now on +/- signs after note letters and numbers following the signs denote intonation in cents). Since we know that perception of the pitch of pure tones (such as harmonics) is influenced by even such parameters as volume, we can safely assume that we can round-off all microtonal intonations to their nearest 5 cent subdivision without loosing much information. This would bring the JI M3 above the fundamental note/reference note to 385 cents and the P5 above the fundamental note to 700 cents. We should mention here shortly, as we will expand on his later, that the rounding off we are using is made possible by our decision to base our hierarchy of intervallic relations on how the ear processes the overtone structure(s) of a tone. If we had tried to create our hierarchy based on the theory that the ear has a preference for abstract simple ratios the P5 and the JI M3 intervals would again be on top of the hierarchy but we would not have been able to do any rounding-off.

Having established the beginning of our hierarchy of intervallic relations we need to also start building the appropriate pitch space representation of this hierarchy. A structurally correct and useful pitch space should clearly show the hierarchy of ties that exist between pitches. Thus a pitch space should place immediately next to a pitch, (i.e. C) its fifth and its major third, (i.e. G and E-15). Since we have two intervallic qualities to relate to one note, we need to use two dimensions for our pitch space giving one dimension to the fifth (i.e. the horizontal) and the other dimension (i.e. the vertical) to the third. Thus, an accurate representation of the overtone produced major triad collection, the representation of the primary pitch/intervallic relationships of a note, should look like this:

\[
\text{Primary collection} \\
\begin{array}{cc}
E-15 \\
C & G \\
\end{array}
\]  

(figure 1)

Since we are representing the primary pitch/intervallic relationships of a note we call the resulting representation the Primary Collection.
We can show all the intervals of the Primary Collection, all the distances between the pitches, by connecting the pitches of the space and creating the shape of an orthogonal triangle.

![Orthogonal Triangle](image)

(Figure 2)

We now see that the m3 that results between the P5 and M3 of the triad can be very accurately represented by our P5 and M3 axes as the result of the combination of one motion on the P5 axis and one motion on the M3 axis. The actual interval is shown as a diagonal connecting the P5 and M3 axes.

We now have a full representation of the primary pitch/intervallic relationships promoted by the overtone structure of a complex tone. Since all relationships being discussed here are strong, two and three way relationships, by establishing the primary relationships of one note we have also established the secondary relationships of the other two notes of the collection. The primary collection shows the primary relationships of the note serving as the root of our major triad and the secondary relationships of the notes serving as the third and the fifth. By generalizing these relationships for the note serving as the root (the note C in our example), in other words by creating three major triads and giving C a different position (root, third, fifth) in each one, we can conclude that all pitches are primarily related to pitches of their own major triad and secondarily related to pitches of the major triads a P5 and a M3 below them. Thus a collection containing both the primary and secondary pitch/intervallic relationships of a note (i.e. C) would contain the P5s above and below that note (G, F), the JI M3s above and below (Ab+15, E-15) and the JI m3s above and below (A-15 and Eb+15). Since the intervals for the creation of this collection have been extracted from the overtone structure of a complex tone, which is a simultaneity, the primary/secondary relationships we are discussing are of a harmonic origin (as opposed to a melodic origin). We should therefore call this collection the Harmonic Primary/Secondary Collection. We can represent the secondary pitch/intervallic relationships by simply extending the three axes established earlier by the primary relationships (horizontal for the fifths, vertical for the M3s, diagonal for the m3s). The beginnings of a P5s/JI M3s space will appear.

*Harmonic primary/secondary collection (P5s/JI M3s space)*

<table>
<thead>
<tr>
<th>A-15</th>
<th>E-15</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>C</td>
</tr>
<tr>
<td>Ab+15</td>
<td>Eb+15</td>
</tr>
</tbody>
</table>

(Figure 3)
Since the harmonic primary/secondary collection originates from a major triad collection, its representation is bound to form a P5s/M3s space. The P5s/M3s space is a fully consistent, coherent extension of the pitch/intervalllic relationships established by the major triad. All relationships shown in the above collection can be traced to the triad. The original triangle representing the major triad can be used to show all the triadic relationships that the reference note is/can be part of. The three orthogonal triangles with the 90° angle on the lower left-hand corner are the three major triads that involve C in a different role. The central triangle shows the primary relationships, the side triangles the secondary. The three (inverted) orthogonal triangles with the 90° angle at the upper right, are the three minor triads that involve C.

The claim that the above collection (figure 3) contains the primary and secondary harmonic pitch/intervalllic relationships of a note is corroborated by both consonance and tonalness ratings. In consonance ratings obtained by Nordmark and Fahlen [1988] the octave, the fifth, its inversion (the P4) and the M3 are clearly judged to be the most consonant intervals. They are followed by the major sixth, the minor 3rd and the minor sixth. As John Pierce [1998] points out, this should come as no surprise since one of the keys to consonance ratings is the overlapping of harmonics. The octave, the 19th (the P5 in an octave generalized space), the double octave and the 16th (the M3 in an octave generalized space) clearly produce the strongest alignment of harmonics. In an octave generalized space these intervals form our familiar primary major triad. Two more octave generalized, superconsonant collections can be created with C in them and they are the major triads expressing the secondary pitch/intervalllic relationships of C.

The harmonic primary/secondary collection would also remain the same if we were to choose intervals based on their calculated tonalness. Parncutt [1989, p. 139] defines pure tonalness as depending on “the number and audibility of a sound’s pure tone components” and complex tonalness, as depending on “the audibility of a sound’s most audible complex tone component”. (For example, a chord that allows the harmonics of its constituting notes to combine to promote one of its tones has a higher tonalness than a chord where all the notes have the same audibility/prominance). Complex tonalness is a much more relevant rating when dealing with polyphonic musical passages [Parncutt 1989, p. 142]. In general, consonance ratings and calculated tonalness follow a similar order [Parncutt 1989, p. 412] and would thus produce the same collection of harmonic primary and secondary intervalllic relationships for a note. Parncutt’s list of calculated complex tonalness of dyads, has the octave first, followed by the P5 and the M3. Once more we see the intervals of our primary collection play an important role in establishing the identity and strength of a pitch. Tonalness is mentioned here because it translates consonance to pitch prominence and can thus help us decide pitch prominence in our microtonal composition examples that will close this paper.

So we see that the overtone structure, regardless of whether the criteria are prominence of tones in pitch perception or consonance, establishes the major triad as the collection of a note’s primary relations and that in turn establishes an extended collection that includes both the primary and secondary harmonic relations of a note. Because the major triad is
the beginning point of our relationships hierarchy, the P5s/M3s space appears as the natural pitch space representation of this developing hierarchy.

It is of the greatest importance to point out that by establishing a collection of the harmonic primary and secondary relationships of a note (i.e. C) we are also establishing a collection that strongly supports and promotes that note. The notes of the primary collection promote C by enhancing/corroborating its overtone structure. The notes of the secondary collection enhance with their audible overtones the overtone structure of one of the notes of the primary collection. Since both the primary and secondary collections grow out of and enhance the overtone structure of C it should be expected that C has a high chance of moving to or being combined with the notes of these collections and that, if handled correctly, these collections will point back to C as their origin and center. The primary and the harmonic primary/secondary collections give us a hierarchy that leads safely away from and safely back to the note of origin. We can thus now begin to refer to the note of origin of our octave generalized P5s/M3s space as the central/reference note and also as the gravitational center.

The overtone based relations that make up this hierarchy are expressed much more accurately and the corroboration/alignment of harmonics is much stronger when using the intervals of a P5s/JIM3s space (figure 3) than the intervals of a P5/ETM3s space (figure 4). (As show below in the P5s/ETM3s space all M3s would be 400 cents and all m3s 300 cents)

**Harmonic primary/secondary collection (P5s/ETM3s space)**

<table>
<thead>
<tr>
<th>A</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>C</td>
</tr>
<tr>
<td>Ab</td>
<td>Eb</td>
</tr>
</tbody>
</table>

(Figure 4)

We must therefore conclude that the P5s/JIM3s harmonic primary/secondary collection can define and support its origin and center of gravity better than the same collection on a P5s/ETM3s space. So although we tend to associate pitch hierarchies with the diatonic system that is a special subset of the 12 note ET system, we see here that the microtones of a P5s/JIM3s space enhance and clarify hierarchical pitch structure in ways that a P5s/ETM3s space can not (we will return to this). The P5s/JIM3s space allows us to view JI not as an abstract application of low number ratios but as a practical application that achieves greater consonance and tonalness and consequently allows us to sharply focus and define our pitch hierarchies.

The harmonic primary/secondary collection does not include all the primary and secondary relationships of a note. When deciding the strength of the relationship of two sequential notes (as in a melodic passage) we also need to consider the influence of pitch distance. In general, the smaller the distance the stronger the relationship [Parncut 1989, p. 61]. In our 12 semitone oriented culture the smallest pitch distance available is the semitone. Therefore, in terms of pitch distance, the reference note of our pitch space (i.e. C) is closely related to its semitones above and below and we need to extend our
collection of primary and secondary relationships to include these two notes. The P5s/M3s space places the semitones above and below our reference note exactly where they should be: one move away from the reference note on the diagonal axes (the one not used by the m3). All other pitches strongly related to the reference note are also just one move away from the reference note. It is thus clear that the P5s/M3s pitch space is ideal for showing the hierarchy of the pitch/intervallic relationships of a note. Our updated collection, that actually includes all the primary and secondary pitch/intervallic relationships of a note (from now on primary/secondary collection), looks like this:

<table>
<thead>
<tr>
<th>Primary/secondary collection (P5s/JIM3s space)</th>
<th>A-15</th>
<th>E-15</th>
<th>B-15</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>F</td>
<td>C</td>
<td>G</td>
</tr>
<tr>
<td></td>
<td>Db+15</td>
<td>Ab+15</td>
<td>Eb+15</td>
</tr>
</tbody>
</table>

(Figure 5)

The resulting intonations of the m2s in a P5s/JIM3s space guarantee high consonance for dyads and triads that involve the m2s. As expected the intonations of the m2s are also close to JI (4 cents or less). However the m2 diagonal axis shows the relationship of pitches in terms of distance and not in terms of common overtones or consonance. It is fine to use JI for pitches found on the m2 axis when only working towards consonant chords. However, when using the m2 axes to express closeness in terms of pitch distance we might want to use ET m2s or even smaller m2s. For example, when writing a diatonic sequence that involves a m2 motion from the leading tone to the reference note/tonic we might want to pull the leading tone closer to the reference note to point to the gravity pull of that note [Parncut 1989, p. 61]. We will see later that our space will allow us to do this extremely effectively and within the confines of our structures.

Since we are living in the western world we might feel that the above collection does not yet include all the important pitch relationships of a note. Because of our continuous exposure to the diatonic system we relate a note with all the notes of its diatonic collection. We can get a collection that includes all diatonic notes of C by simply adding D (the M2 above the reference note) to our primary/secondary collection. The M2 above is placed two steps away from the reference note and that shows that the M2 above is a more distant relative of the reference note than the primary and secondary relationships already expressed. The most appropriate placement for the D is the P5 axes (one step past the G). Since it was our concern for the inclusion of the full diatonic scale that prompted the addition of the D, we should not place the D on the m2 axes as this would mediate the move from C to D through a non diatonic note (C#). Also this placement relates the D to the C through the note most strongly related to C (G).

Our collection of all the primary, secondary and diatonic pitch/intervallic relationships of a note (from now on also the primary/secondary/diatonic collection) looks like this:

<table>
<thead>
<tr>
<th>Primary/secondary/diatonic collection (P5s/JIM3s space)</th>
<th>A-15</th>
<th>E-15</th>
<th>B-15</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>F</td>
<td>C</td>
<td>G</td>
</tr>
<tr>
<td></td>
<td>D</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Db+15</td>
<td>Ab+15</td>
<td>Eb+15</td>
</tr>
</tbody>
</table>
It is fairly easy to notice that if we include the JIM3s above and below the D we get an instance of each of the 12 semitone classes. We thus create a 12 note P5s/JIM3s space showing as accurately as possible the hierarchy of relationships between the central/reference note and the other 11 semitone classes. All intervals involving the reference note are 4 cents or closer to JI.

12 note collection, C centered (P5s/JIM3s space)

<table>
<thead>
<tr>
<th></th>
<th>A-15</th>
<th>E-15</th>
<th>B-15</th>
<th>F#-15</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>C</td>
<td>G</td>
<td>D</td>
<td></td>
</tr>
<tr>
<td>Db+15</td>
<td>Ab+15</td>
<td>Eb+15</td>
<td>Bb+15</td>
<td></td>
</tr>
</tbody>
</table>

However, the above 12 note space is not a closed space. The notes of the P5 axes can not form a circle of fifths. F is not 700 cents above Bb+15 and A-15 is not 700 cents above D. (This is happening because the higher and lower P5 axes have not been created through 12 sequential ET P5 motions but through JI M3 motions up and down from the central P5 axis). Furthermore, none of the M3 axes form circles since two JI M3s up do not give us the same note as one JI M3 down. The above space is an open space. It shows very accurately the hierarchy of pitch/intervallacl relationships of its central note but can not show accurately the pitch/intervallacl relationships for all its notes. Every change of central/reference note would need to introduce new notes in order to show its hierarchy. This would result in a pitch space with many more than 12 notes. The space would extend outwards indefinitely. This will take us far away from the established 12 semitone ET system. It would also make it impossible for us to create a compact finite pitch space representation that would allow the composer to see all possible relations of the system at once. Some of the original decisions we made for the creation of our practical microtonal system seem to be incompatible. Using an accurate representation of the pitch perception based hierarchy of pitch/intervallacl relations as the main structure of our system gives us hierarchy enhancing microtones but seems to takes us away from a compact, economical pitch space and from the predominant pitch system of the western world. As we pointed out earlier, there are nagging questions that do not allow us to treat the above incompatibility as an accuracy vs economy question. We showed that our microtonal P5/JIM3s space supports pitch hierarchy better than a P5s/ETM3s space. If this is the case then why is the diatonic system, that aims to create strong pitch hierarchies, so strongly tied to ET? Is it possible that the pitch perception based hierarchy of intervals promotes at the same time use of microtones and a 12 note ET subdivision of the pitch space?

The reasons for a 12 note ET P5s/ETM3s space and the benefits of such a space

We know that the perception based hierarchy of intervallacl relationships promotes a P5s/M3s pitch space representation. It turns out that a P5s/M3s space promotes in turn the subdivision of the pitch continuum into 12. In his wonderful paper, “The Group-theoretic description of 12-fold and Microtonal Pitch systems” Gerald Balzano [1980]
shows that there are binding mathematical connections between a P5s/M3s pitch space and the subdivision of the pitch continuum into 12 semitones.

Balzano shows that if we are to structure a pitch space of P5s and M3s, mathematical consistency would promote a 12 semitone equal temperament representation as the natural third representation of this space and vice versa. We have shown earlier that sensory reasons make the triad the originating structure of the P5s/M3s space. Balzano shows that using the triad as a basic structural unit mathematically promotes the 12 semitone subdivision of the pitch continuum. Balzano also shows that the 12 semitone ET P5s/M3s space and the diatonic scale are mathematically intertwined. The diatonic scale is a unique collection of this pitch space and it (the diatonic scale) brings out “the more abstract relations of a fifths and thirds space while remaining coherent with respect to semitones”. Lastly, Balzano points out that the P5s/M3s pitch space allows us to monitor many important relations of the ET system and of its diatonic subsets:
- melodic closeness on the semitone axes
- relations of triads show by the points and edges shared by the triangles representing the triads
- closeness of diatonic scales on the P5 axes

To create a 12 semitone ET P5/M3s space all we need to do is abstract the microtonal intonations from our pitch perception based 12 note collection (figure 7) by using ET M3 axes instead of JI M3 axes.

12 note collection, C centered
(P5s/ETM3s space)

\[
\begin{array}{cccc}
A & E & B & F# \\
F & C & G & D \\
Db & Ab & Eb & Bb
\end{array}
\]

(Figure 8)

The benefits and costs of a P5s/ETM3s space are fairly obvious. By choosing to ignore microtonal tunings we can create the abstract notion of a semitone class. We can then use the 12 semitone ET P5s/M3s space as an all inclusive and fairly accurate representation of the pitch relationship hierarchies of all the 12 semitone classes of the ET system. All the M3 axes are circles and the three P5 axes, written sequentially with each one being read from left to right, give us another circle; the circle of fifths. This is therefore a closed system and all possible relationships between all notes are included within. The total inclusiveness and economy of the closed 12 semitone ET system along with the mathematical consistency reasons offered by Balzano form a strong battery of justifications for the abstraction of microtones that allowed the creation of ET, the dominance of 12ET and the strong connections of the diatonic system to 12ET.

One final reason that supports the division of our pitch space into 12 categories is given by Parncutt [1989, p. 64]. A 12 categories pitch space allows us to easily assign a category to all distinguishable overtones, even the out-of-tune ones, of a complex tone. We can thus easily build perceptual representations of the overtone structures of complex tones and use these perceptual representations to quickly parse a musical environment. If
our pitch space was a quartertone space with 24 categories, assignment of out-of-tune overtones to a distinct category would have been very difficult making parsing of complex musical environments hard.

Combining JI and ET into a closed, powerful, two layer system

Having established the arguments for the use of a P5s/JIM3s space and the arguments for a P5s/ETM3s space we are faced with an age long question. How can we include the benefits of finely tuned JI relations without loosing the benefits of the closed 12ET system? The solution is to look for a way to include in the 12 note ET space the structure enhancing information provided by the JIM3 axes without allowing the space to lose its structural coherence and functionality. By treating 12ET as a higher level abstraction and creating a second level that includes only the structure enhancing microtonal intonations provided by the JI M3 axes we can create a two layer system that can achieve this powerful combination.

We first need to establish which intonations provided by the JIM3 axes are structure enhancing and which are not. While building the representation of our pitch perception based hierarchy of intervals we showed the structure enhancing role of intonations that lie one JI M3 up or down from the central P5s axis. We will now see that enlarging our space by allowing the JI M3 axes to extend two steps away (two JI M3s up or down) from the central axis will also provide structural benefits. This extension will result in a 20 note collection centered around our original central note.

Unfolded, 20 note, C centered, P5s/JIM3s collection/environment

<table>
<thead>
<tr>
<th></th>
<th>C#</th>
<th>G#</th>
<th>D#</th>
<th>A#</th>
</tr>
</thead>
<tbody>
<tr>
<td>C-30</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A-15</td>
<td>E-15</td>
<td>B-15</td>
<td>F#-15</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>C</td>
<td>G</td>
<td>D</td>
<td></td>
</tr>
<tr>
<td>Db+15</td>
<td>Ab+15</td>
<td>Eb+15</td>
<td>Bb+15</td>
<td></td>
</tr>
<tr>
<td>Bbb+30</td>
<td>Fb+30</td>
<td>Cb+30</td>
<td>Gb+30</td>
<td></td>
</tr>
</tbody>
</table>

(Figure 9)

We will call this 20 note collection also an environment because a) it is the first one that includes duplicate versions of semitone classes and b) it is our last collection and as such includes all the previous ones. Let us look at the structural benefits provided by this extended collection/environment.

Assume we are working in a C centered space (like the 20 note collection above). We play the center note (C). Its overtones sound E-15. If we now play the E-15 its overtones will sound a G#-30. However, G# is also Ab. Ab is part of the primary/secondary collection of C and its overtones should support the central note C by aligning the M3 above Ab with the C. For this to happen we need to use a G#/Ab+15 (385 cents below C). So if the next note we want to play (after the E-15) is a G# (or Ab) should we write G#-30 or G#/Ab+15? Fortunately the answer is given by the question. If we move from...
C to G# through E then we will write a G#-30. If we move to G# directly from C then we will write G#/Ab+15. The 20 note environment enriches our microtonal vocabulary while still allowing us to write sequences that achieve alignment of overtones. At the same time we are using microtonal intonations to show the exact course of the relationship between pitches (to get a G#-30 in a C centered environment it means that we went through E-15).

The 20 note collection/environment includes the JI tunings, at an approximation of 4 cents or less, for all intervals starting at each one of the notes of the 12 note collection of the same center (in the case of a C centered environment, the 12 note C centered P5s/JIM3 collection shown in figure 7). This means that we can create a JI 12 note collection around each one of the notes of the central 12 note collection and we can modulate from the central 12 note collection (the C centered 12 note collection) to any of the other eleven (non-central) 12 note collections. These (inner-environment) modulations however, would not create as complete a change of focus as the modulations we see in 12ET. The 20 note collection/environment, like the collections at the previous 5 levels of our hierarchy, grows from the overtone structure of the central note and is unique for each original central note. All the notes and thus all collections of an environment originate from the central note and gravitate to it. The central note of an environment always retains an overall influence. Therefore a JI 12 note collection that is not built around the central/reference note of the environment (a non-central JI 12 note collection; i.e. a JI 12 note collection built around the note A+15 of a C centered environment) will denote two centers of gravity: a local center (its own central note, i.e A+15) and an overall center (the central/reference note of the environment, i.e. C). Since all notes of the environment gravitate to the central note we can assume that the existence of a local center of gravity other than the overall central note would only be temporary as the overall central note will eventually enforce itself. We can therefore distinguish between two kinds of centers of gravity and two corresponding types of modulation. We will call a local center of gravity a “point of emphasis” and we will save the term “central/reference note” for the overall central note of an environment. We will call inner-environment modulations (modulations within an environment) “temporary change of point of emphasis” and we will save the term “modulation” for an overall change of central/reference note (for a change of environment). A further development of our system that will take place later will enhance the differentiation between temporary change of points of emphasis and modulation. The 20 note environment is clearly a structure enriching tool as it allows the composer to establish two levels of centers of gravity and corresponding modulations. It also allows the use of intonation to achieve alignment of overtones and strengthen a center, show relationships and imply progressions.

The unfolded 20 note environment places two microtonal intonations of the same semitone class at two different locations of the pitch space. By doing so it fails to clearly show that we are dealing with two variations of the same semitone class. It fails to show the intonation choices that the composer has to make as we discussed them above (in the G#/Ab example). Since our space uses octave equivalence, even if two versions of a semitone are reached by sequences of intervals that place them in different octaves it
should not matter. Microtonal variations of the same semitone class should occupy the same vicinity so that their proximity can be shown. This approach will allow us to achieve the compact, practical kind of space we set out to create. Where the composer in one glance can be aware of all the possible relations for a note and can trace each relation to a specific hierarchy. We will therefore fold the space of the environment. In the folded space we will use only one spelling for each semitone class. Next to each semitone class we will write all the possible intonations it can have in the given environment. This will show microtones as variations of the 12 semitone classes which will facilitate the structuring of our final closed, two layer, space (see figure below and related discussion) and will allow us to hierarchize the alternate spellings and intonations for each note.

Since the whole pitch space originates from the JI major triad collection (a JI M3 and a P5) and in that context JI M3s are much stronger relationships than diminished 4ths, we will choose to express all M3 relationships as such and not as diminished 4ths. When laying out note relationships in a pitch space, diminished fourths that may result from two JI M3 motions up or down from the central P5 axis will be expressed as M3s. For example, in a C centered space a G# -30 will be show as Ab –30. However, in a composition score, where all notes are in context, the composer should choose the spelling that clearly shows the progression that preceded a note. For example, if we move from C directly to Ab we should notate the resulting note as Ab+15. If we move to Ab/G# through E we should write G#-30 (C, E-15, G#-30, two sequential M3s). Here is the folded version of the 20 note collection/environment.

20 note, C centered, P5s/JIM3s collection/environment

<table>
<thead>
<tr>
<th>+30</th>
<th>+30</th>
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<th>+30</th>
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<tbody>
<tr>
<td>A -15</td>
<td>E -15</td>
<td>B -15</td>
<td>F# -15</td>
</tr>
</tbody>
</table>

F     C     G     D

Db+15     Ab+15   Eb+15   Bb+15

-30         -30         -30         -30

(Figure 10)

So extending our space by allowing two sequential JI M3 moves away from the central P5s axis is beneficial as it enhances our structural vocabulary. However, enlarging our space further by allowing the JI M3s axes to extend 3 steps up or down from the central axis would not be beneficial. Three sequential JI M3s motions away from the central axis, in a space like the C centered space we are working in, will create –45 + 45 intonations (like C-45). These intonations will challenge the original central note (C) and the central axis of the space. (the F-C-G-D axis). One of the benefits of the P5s/JIM3s space is that it enhances our ability to create structures with a clear gravitational center. Therefore it would not be consistent for the system to allow the chains of JI M3s to come around and produce an intonation (C-45) that will challenge the central note. The +/- 45 intonations would also emphasize, or give a reference role, to the +/-30 cent intonation notes. These notes have no direct, overtone based connection to the central note C and thus emphasizing them will not help our hierarchy. Furthermore, our system should not allow any +/-45 tunings because they are too close to quartertone intonations. Quartetone
intonations fall at the half-way point between semitones. It has been show that the categorical perception promoted by the use of 12 semitone ET gets weaker as we move away from the ET tuning of each semitone and is at its weakest around the half-way point [Burns and Ward 1978]. This means that the listener might have a hard time categorizing a quartertone as an ET note that is either sharp or flat since the listener might feel that a quartertone is a distinct tone that does not belong to any of the 12 categories of the ET system. Thus the wide use of quartertones might promote the appearance of 24, finer, quartertone-size categories. We do not want our microtonal pitch space to be a 24 (or more) tone space. Because of sensory and structural reasons we want microtones to simply enhance a 12 semitone space. For all above reasons we conclude that the JI M3 axes can only extend two motions up or down from the P5s axis of the central note (the central axis) without challenging the coherence, structure and functionality of our pitch space. We also conclude that the largest collection that can be built in reference to and in support of a central note is the 20 note collection /environment. We have now partially limited the openness of our P5s/JIM3s space while including crucial microtonal information. However, we can not yet claim that our space also includes the important benefits of a closed space or of 12ET.

The solution that will close the space and bring 12ET into it is simple. Since in our microtonal space we chose to round off the JI P5s into ET P5s it is possible to extend and fold the central, 4 note, P5s axis of our 20 note collection/environment so as to include in our space the 12 note ET circle of fifths (in other words we will add the ET intonations for the notes of the higher and lower P5 axes of our 20 note collection). Throughout the above discussion we have used an ET note (ET C) for a central/reference note. In order to achieve the correct intonation of intervals we moved the notes surrounding the reference note away from ET and allowed the reference note to retain its ET. One of the main benefits of the 12 note ET circle of fifths is that it is a closed system with each of its notes able to be a center (or tonic). Therefore if we add the ET circle of fifths to our pitch space and choose to allow only the 12 ET notes of the circle of fifths to be central/reference notes we will be including in our microtonal pitch system the benefits of ET at the most important level, the level of central notes (or tonics). Furthermore, since only notes on the three, 4 note, ET P5s axes will be central notes and our space allows only up to two consecutive 385 cent motions up or down from the P5s axis of the current central note (or in other words because the largest collection that can be built around a central note is a 20 note environment) and since in our pitch space we use only one spelling for each semitone class, our space will have only 12 note classes and each note class will have only 5 possible intonations: ET when the note is on the P5s axis of the current central note, -15 cents when the note results from one JIM3 motion upwards from the central axis, -30 cents when the note results from two consecutive JIM3 motions up from the central axis, +15 cents when the note results from one JIM3 motion down from the central axis, +30 if the note results from two consecutive JIM3 motions down from the central axis. The total amount of notes will be fixed (12X5=60). This will be a closed system in terms of gravitational centers (12) and a finite system in terms of number of notes (60) and microtonal inflections (4). These 60 notes will be adequate to give us all JI tunings, at an approximation of 4 cents or less, for all 20 notes of each of the 12 environments allowed by the system (one environment for each ET semitone). With these 60 notes we can build
the microtonal pitch space hierarchy we built for C for each of the 12 ET semitones. The resulting space will be a practical, successful combination of the compatible benefits of JI and ET.

### P5s/JIM3s, 12 ET center, microtonal pitch space

<table>
<thead>
<tr>
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<td>A</td>
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<td>+15</td>
</tr>
<tr>
<td>B</td>
<td>-30</td>
<td>-30</td>
<td>-30</td>
<td>-30</td>
</tr>
<tr>
<td>F#</td>
<td>+30</td>
<td>+30</td>
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<td></td>
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<tr>
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<td>D</td>
<td>+15</td>
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</tr>
<tr>
<td>Ab</td>
<td>-30</td>
<td>-30</td>
<td>-30</td>
<td>-30</td>
</tr>
</tbody>
</table>

(Figure 11)

Let us quickly review some of the important benefits of the above space. In this 12 center setup we can use intonation to denote the role of a note. When a note uses ET we are showing it as a possible center. When a note is not using ET tuning we are indicating that this is not a central/reference note --at least at the environment level-- and that its intonation is the result of the influence of the current central ET note (the current overall center of gravity). Intonation can even indicate the possible central note(s) that is/are causing the microtonal intonation and even the progression that got us to that intonation. For example, an E-15 is most likely a M3 above a C reference note or a m3 below a G reference note. A G#-30 is most likely the M3 above the M3 above the central note of a C centered collection (the M3 above E-15). Compare this clarity and uniqueness of relationships achieved through microtones with the number of possible relationships of a note in a 12 semitone total ET system. For example, an E of a 12 note ET pitch space can have 10 different roles/functions in tonal (major-minor) settings. Our P5s/JIM3s, 12 ET center, pitch space establishes unique, structure revealing relationships between notes. Furthermore the space produces unique collections of notes at all 6 levels of our collections hierarchy (primary, harmonic primary/secondary, primary/secondary, primary/secondary/diatonic, 12 note collection, 20 note collection/environment) for all 12 central/reference notes. These unique relationships and collections can be a powerful tool for denoting active hierarchies in musical passages. The microtones of the JIM3 axes do not simply denote and reveal the active hierarchies. They actually enhance them. These JI tunings can promote through corroboration a unique gravitational center for and increase the consonance and stability of a collection or passage. (It obviously follows that
contrasting intonations can be used to destabilize a passage). Our space also allows us to differentiate between central notes and temporary points of emphasis and corresponding levels of modulation in a composition. We can either have a definite, overall change of central/reference note (a change of environment) or a temporary, local, change of emphasis within an environment. To clearly separate the two, we reserve the word modulation for an overall change of central note and we use the term temporary change of point of emphasis for changes within an environment. During a modulation our center has to move from one ET note to another ET note. During a temporary change of point of emphasis any of the 11 non-central notes of a 12 note collection can become a temporary point of emphasis while the listener remains aware of the overall central note (of the ET central/reference note of the environment being used). This is possible because our space includes all JI tunings, at an approximation of 4 cents or less, for all possible 12 note collections within each of the 12 possible environments and because each of those collections and environments is unique. By looking at the m2 axes we realize that each of the 12 semitone classes in our space have 5 possible intonations. This allows us to use the m2 axes to finely express relationships in terms of pitch distance. Neighboring semitones can be brought as close as 70 cents to show the gravity pull of one note on another (tonic pulling the leading tone for example) and as close as 40 cents to express mutual attraction of neighboring notes or the gravitational pull of competing centers. Neighboring semitones can also be brought apart as far as 160 cents to express repulsion or again the influence of competing centers. Because of the 5 available intonations for each note the system has a large number of microtonal intervals (for every interval there are 9 possible expressions). They allow for powerful microtonal passages with each move denoting a possible intention. Denoted intentions can be in agreement to stabilize and focus a passage or be contrasting and thus create ambiguity. We will see both kinds of results in the music examples below.

A short addendum on the rounding off choices and on exact ratios

Preference for exact rations has been a popular way of establishing the P5s/M3s space for centuries. In this paper however, since we are trying to create a practical system, we have chosen to stay away from arguments that are interesting at the theoretical level but move us away from obvious practical benefits. The first important benefit of the rounding off of JI P5s and JI M3s used in our space is that it allows us to create and include in our microtonal space the closed circle of 12 ET P5s thus connecting our JI produced microtonal vocabulary with 12ET. The second benefit is that all microtonal variations of our system are provided by the rounded off JI M3s. Therefore all microtonal intonations are divisible by 5, establishing 5 cents as the smallest subdivision that our system has to express. A system using exact ratios for the JI M3s (386.3 cents) would have to use 0.1 cents as the minimum subdivision.

Furthermore, our approach, based on pitch perception processes, allows us to assume that at each instance the sounding note supplies the tuning information for the following P5 or M3. Thus we do not have to worry about summing the effect of this rounding off when writing sequences of P5s and/or M3s. Since the resulting note of each step of the sequence provides the information for the next step and since at each step the rounding
off is non-perceivable it follows that our ear will not perceive a sum effect of the rounding off and that our sequence will sound correct. (For example, in the sequence F-C-D, the overtones of F will provide a C+2. The rounded off C+0 will sound correct to the ear and its overtones will be called upon to provide the G. This will be a G+2 that will be rounded off to G+0.)

**Notation, intonation exercises and introduction to the music examples**

Since only two types of microtonal intonations are allowed in our system (+/- 15, 30) and the second one is twice as large as the first, when writing a score we can choose the sign of an arrow for the 30 cent intonation, turn it up or down to denote +/- and half its value (through a flag) to denote the 15 cent intonation (see also music examples).

![Figure 12]

Although our proposed pitch space rises naturally out of sensory processes and mathematics that guarantee its structural validity, the dominance of total ET and the expectations this dominance has created might make it hard for some performers or listeners to hear and/or perform the proposed refined structures. Since our microtonal intonations originate from the JI M3 axes, certain intonation exercises focusing on JIM3s might assist the preparation of performers. Over a sustained tone, performers might be asked to perform JI M3s above and below with the aim of reducing the amount of beating/harshness as much as possible. This will point the ear to the consonance origin of the JI M3 and assist the performer in achieving the -/+ 15 intonations. Further exercises with sequences of two JI M3s will assist the performer in the accurate rendering of the +/-30 intonations.

You can listen to all the music examples given below by downloading them from http://isa.hc.asu.edu/~thanassis/micropap/soundexamples.html. Computer generated tones are used for the realization of the music examples. All tones are complex tones with only the lower 10 harmonics present as those are the audible harmonics that have helped us form the base of our system. Each harmonic “n” is given an amplitude of 1/n since that is a pretty accurate representation of the audibility of each harmonic. Relative amplitudes for each harmonic are given below:

<table>
<thead>
<tr>
<th>Harmonic number n</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch</td>
<td>C4</td>
<td>C5</td>
<td>G5</td>
<td>C6</td>
<td>E6</td>
<td>G6</td>
<td>Bb6</td>
<td>C7</td>
<td>D7</td>
<td>E7</td>
</tr>
<tr>
<td>Amplitude</td>
<td>1</td>
<td>.5</td>
<td>.333</td>
<td>.25</td>
<td>.2</td>
<td>.166</td>
<td>.142</td>
<td>.125</td>
<td>.111</td>
<td>.1</td>
</tr>
</tbody>
</table>

![Figure 13]
and the actual content of the music examples do not aim at exemplary compositional results. They only aim to show the applicability of our claims regarding this pitch space. For an actual compositional example please go to http://isa.hc.asu.edu/~thanassis/works and download the first movement of the work Krisis for cello, trombone, flute and bassoon.

Music Examples

Examples 1 and 1a

Example 1/1a creates a driving microtonal melisma by toying between tendencies created by intonation relationships from the P5 and JI M3 axes (the harmonic relationships of B to G and D) on the one hand and intonation relationships from the m2 axis (the higher and lower semitones of B) on the other. Note how easily the ear picks up the microtonal changes and follows the different tendencies they generate.

For the last chord (dyad) of example 1 we use ET C and ET Eb. For the last chord of example 1a we use the dyad ET C, Eb+15. The JI Eb of example 1a fits as well as possible with the C (they share G as a common overtone) and is also the correct Eb for a G centered collection. The JI Eb corroborates the rest of the cues of the passage that show a progression from G to C in the context of a G centered collection. Thus, there is no strong urgency for an immediate move away from the C,Eb+15 dyad, the passage can temporarily rest on this dyad. In contrast, the ET Eb of example 1 does not fit as well with the overtones of C and does not belong to a G centered collection. Thus, the ET Eb introduces a slight feeling of ambiguity. Our ear has two choices. It can treat the ET Eb as a decorative harshness resulting from the continuous melodic play on the m2 axis that characterizes this example. Alternatively, or in parallel, our ear can choose to treat the Eb as a possible implication of a modulation especially since it coincides with a change
in harmony. In either case a need for resolution is established and we therefore expect the C, Eb dyad to quickly move to another chord.

Our ability to follow the microtonal variations and changing tendencies in the melisma and distinguish the differences between the final chords of examples 1 and 1a shows that even after a very short passage our ear has already learned to recognize the harmonic and melodic differences caused by the microtones of our pitch space and assign meaning to them. This goes to show that if a musical element (like our 15 and 30 cent microtonal variations) is used extensively in a passage then the ear will immediately pick up on its importance. The microtones of our pitch space, when used systematically and consistently, are indeed perceivable and informative at many levels.

In Example 1 we use our microtonal vocabulary to embellish a standard progression. In Example 2 the use of our microtonal vocabulary goes beyond just embellishment as it also helps us prepare a more adventurous progression. Thus we examine the resilience of the microtonal vocabulary under less predictable circumstances. Will the microtones of a melisma be as acceptable if they point to a non-diatonic melodic relationship from the m2 axis (III to flat III) rather than the usual diatonic relationship of the third degree rising to the fourth? Will the microtonal line sound as convincing if it drives a less familiar, less expected I-VI progression, that comes from the M3 axis, instead of a very familiar I-IV progression which is part of the dominant P5 axis?

Example 2

Through downward motions that get longer and lower the microtonal melisma prepares the move from B to Bb. The emphasis on B as a note with some individuality also promotes the overtones of B and brings into play Eb before its actual appearance. The resulting microtonal line sounds totally acceptable even though it powers a less familiar progression and gradually moves away from the expected target established by cultural conditioning.

We should point out that example 2 puts emphasis on the two M3s that surround the G (B and Eb). The example sounds acceptable even though, in some sense, it arpeggiates the M3 axis which is an augmented collection. This acceptance of the augmented triad can be partly attributed to the preparation offered by the microtonal line but we should also
consider Parncutt’s point [1989 p. 141] that the perceived dissonance of the augmented triad “might have cultural rather than sensory origins”.

For the final chord of example 2 we use ET for the Eb, a P5 for its 5th (ET Bb) and JI for its M3 (G-15). These choices increase the tonalness of the ET Eb and the consonance of the Eb major chord and point to a modulation away from a G centered collection (a G centered collection would be better supported by an ET G, an Eb+15 (JI M3 below) and a Bb+15 (JI m3 above)). The ET Eb becomes a candidate for new central note. This modulatory feeling at the end of the example is almost unavoidable. Maintaining the G center feeling though the Eb chord would not have been easy. It would have required using a Bb+15 instead of the ET Bb. However, to justify the gravity pull of Bb on B we need for the Bb to have some strength. The Bb can gain strength either by being a reference note (which requires the use of an ET Bb) or by at least being part of the major triad triangle of a reference note. By establishing the JI Eb major triad as a possible reference note triangle we get an ET Bb that has the second most important role in that triangle. Extending the G centered feeling would also have required using an Eb+15. This would have refused us the ET Bb and would have been in contrast with the lower Ebs sounded by the overtones of the Bs of the microtonal line. The intonation choices for the last chord of example 2 provide the desired result. The unconventional progression is justified by the implied gravitational pull of the ET Eb and Bb.

In example 3 we present the kind of microtonal choices we should make if we want to use temporary points of emphasis (besides the central note) while remaining in the same environment. In other words, we present the kinds of choices we should make if we want to use chromaticism without permanently subverting the original central/reference note of a passage. Our microtonal space is ideally suited to this kind of use. It gives us the microtonal alterations that allow any note or chord, chromatic or not, to relate as accurately as possible to and support as well as possible a reference/central note. The original central note can thus maintain its influence and its implied gravitational strength throughout a chromatic passage.

Example 3

Example 3a

http://isa.hc.asu.edu/~thanassis/micropap/ex3.mp3
http://isa.hc.asu.edu/~thanassis/micropap/ex3a.mp3

(Figure 16)
The original reference/central note of example 3 is C. We want to temporary emphasize A while maintaining C as our implied overall reference note. Through microtonal intonations, all notes of the example (chromatic and diatonic) are treated as part of a C centered 20 note collection/environment on a P5s/JIM3s space. E-15 is the JI M3 above C and G#-30 is the JI M3 above E-15. G#+15(Ab+15) would also be a possible intonation in a C centered collection but E would have no role in this relationship. We definitely need to show that our progression is going through E as we want to use an E major chord to move to the A minor chord. As with all notes of the example, the intonations of the A minor chord show the chord as part of a C centered collection. Towards the end of the example, once we take the A away and C and E-15 and then C sound by themselves we are able to confirm that this was not a modulation but a temporary change of point of emphasis. At the end of the progression C is ready to return to its role as the undisputed central note.

Example 3b shows some of the things we might want to change for example 3 to become modulatory (to denote a change of central note).

The use of an ET E on the second beat, and the increased beating and harshness that this creates, begin to imply that we have competing centers and we might be moving away from C. The intonations of the notes of the E major chord reinforce this feeling as they are in correct reference to E rather than C. Finally, the JI major chord build in reference to the ET A confirms that this passage is modulatory and is moving away from the original central note. It is interesting to compare the timbral feel of example 3a, that modulates between centers and thus uses ET extensively, with the timbral feel of example 3, that stays within the same environment. The ET using example sounds brighter and lacks shadows.

Microtonal intonations in examples 3 and 3a are used to increase the clarity, focus and direction of fairly tonal, consonant, unambiguous passages. Example 4 shows that our microtonal vocabulary can also be used effectively in extensively chromatic, dissonant and ambiguous passages. Contrasting microtonal intonations are used to increase the ambiguity of the passage or to imply competing centers at certain points while at other points coinciding microtonal intonations provide hints of resolution and agreement.
Example 4

http://isa.hc.asu.edu/~thanassis/micropap/ex4.mp3

(Figure 17)

Example 4 starts with a G and the next two notes use intonations of a G centered collection. This set up is soon challenged by an A, C+15 dyad using intonations that refer to ET A. The move to an E natural increases the referential role of ET A. The Eb+30 on beat 4 is supposed to act as a surprising jolt. It achieves this function by sounding very foreign both to what precedes it and to what surrounds it. An Eb is hard to associate with an A minor chord and its intonation (Eb+30) does not allow a connection (through voice leading and pitch proximity) to the preceding G.

The D-15 at the end of the fourth beat increases the dissonance but its intonation and the low F that comes in on the fifth beat hint of a possible move towards an ET Bb. On the fifth beat our ear has the choice of grouping F, A, C+15 and Eb+30 into a V7 of Bb. However this choice is weakened by the C disappearing as soon as the F comes in and because the intonations of the notes of this possibly extracted V7 chord are not those of a Bb centered collection (a clear V7 of Bb should be tuned F, A-15 (or A if we want to show the leading tone being pulled up), C, Eb). Thus, the collection at the middle of the fifth beat can at best be seen as a very hazy V7 of Bb.

On the sixth beat the ear might choose to extract the Bb, D-15, F notes and treat them as a primary collection. Treating Bb as a reference note is further encouraged by the A+15 cents implying gravity pull by the Bb. However, the extracted Bb collection is in a weak inversion and the other three notes sounding are too dissonant for the sixth beat to be treated as a resolution.

By the middle of the 7th beat the Bb primary collection is gone as the F has been removed and D is moved to ET. The F# that comes in might entice the ear to group C,D and F# to make an incomplete V of G and create the expectation that we are returning to our original focal note but the intonation F#-30 challenges this approach.
However, this time we choose to resolve this microtonal ambiguity immediately as F#-30 is moved to ET F# and soon after the E is moved to the correct intonation for a G centered collection. In the final measure the return to G is delayed by few quick skips into ambiguity. The momentary distancing of Bb from its JI relationship to G, the momentary lowering of the G, the return to G through a VI chord rather than a V and the momentary dissonance through F provide some decorative jolts before we settle for good into a G major JI triad.

The above short examples show that the microtonal intonations of our pitch space:
-are perceivable when used consistently and correctly (with awareness of the sensory and mathematical procedures that underlie pitch perception)
-allow the composer to realize relationships that enhance pitch structures and functions
-allow us to combine the benefits of ET and JI
-offer us a better realization of the P5s/M3s space
-are indeed easy to use, structural tools that enhance the expressive means available to the western composer